

Quantifying landscape fragmentation

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Abstract. The features of land cover objects in digital raster images are often described by their pattern, connectivity, and fragmentation. While there are many quantitative measures for pattern and connectivity fragmentation is usually described in a qualitative way, and often for a specific species only living in the landscape under study. The notion of fragmentation comprises many aspects of image properties. A daunting task could be the bottom up approach to derive a series of indicators describing all kind of aspects and then trying to summarize them. In contrast, this study suggests a top-down approach, illustrating and comparing the use of three holistic concepts, based on geometric principles only, and resulting in normalized, quantitative fragmentation metrics describing both, the overall degree as well as the spatial distribution of fragmentation on any categorical land cover map. After providing the motivation for using the frameworks of contagion, complexity, and spatial entropy their algorithmic implementation is explained. The performance and features of the proposed three concepts are exemplified on a binary forest mask. Together with a batch-processing option these tools are available within the free image analysis software GuidosToolbox. The user-friendly provision of operational tools for a generic and especially quantitative assessment of fragmentation could contribute to an improved understanding and interpretation of landscape dynamics. Monitoring and especially quantifying the impact of human activities on our landscapes may also facilitate the design of efficient and assessable landscape resource policies and risk assessment studies.

Keywords: Image Processing, Fragmentation, Entropy, Contagion, Complexity.

1. Introduction

Landscape fragmentation has been linked to a number of environmental consequences including physical effects, Saunders et al. (1991), and biological effects such as a decline in biodiversity due to the loss and isolation of the habitat and/or species, Kupfer and Franklin (2009). Landscape fragmentation comprises many different patch aspects, such as the number and typical shape, the inter-patch distance, pattern, connectivity, and patch configuration. While there are many quantitative measures for pattern, McGarigal et al. (2012), Soille and Vogt (2008), Wickham et al. (2010), Vogt (2014), and connectivity, Saura and Torné (2009), Saura et al. (2011), fragmentation is usually provided as a qualitative description for a specific species living in the landscape under study. Yet, a generic description and interpretation of landscape dynamics in general requires a neutral, repeatable and especially quantitative assessment of fragmentation. On a categorical map fragmentation can be described as the spatial heterogeneity, or the spatial composition and arrangement of foreground objects in an image. In addition, fragmentation accounts for the number of objects and the distance between them, hence addressing foreground and background characteristics at the same time. Due to its holistic nature the description of fragmentation may be rather complex when accounting for and especially when trying to summarize its individual components. Moreover, current fragmentation definitions, see Bogaerts et al. (2012) for a comprehensive list of fragmentation definitions, are usually descriptive. Like the well-known definition of Forman (1995): *Fragmentation is the breaking up of a habitat, ecosystem, or land-use type into smaller parcels*, which is not directly suitable to *quantify* fragmentation. In this paper we will focus on three geometric concepts tailored to describe and quantify fragmentation: contagion, complexity, and entropy.

Contagion: Some methods to detect fragmentation use the concept of aggregation, He et al. (2000) or spatial contagion. Li et al. (2012) used a sliding window analysis outlined in Riitters (2000, 2002) to detect transitions from intact forest areas. A similar approach was

applied by Riitters et al. (2012) to detect degradation of forest and grass-shrub vegetation due to its proximity to anthropogenic land uses. In general, contagion can be interpreted as the complement to fragmentation, i.e. areas having low spatial contagion are highly fragmented and vice versa.

Complexity: In computer science concepts of algorithmic information theory, MacKay (2003), can be applied to measure the essential information content of image objects. For a simple example of a text string, the Kolmogorov complexity, Kolmogorov (1998), is the length of the shortest possible representation of the given string using a lossless compression technology. The degree of complexity can be described as the difference of the original to its compressed string length. Using this notion, a boundary condition is found for an incompressible string with an algorithmically random representation. Zenil et al. (2012) suggest using Bennett's concept of logical depth, Bennett (1988, 1990), as an improved version of the Kolmogorov complexity. While the latter measures string length compression size the logical depth uses decompression times to estimate complexity. Here, the idea is that time-consuming decompression is directly related to a high degree of complexity/information and fast decompression is found for trivial shaped objects or a random distribution, where the compression and decompression algorithm is straightforward. Although examples show that the logical depth might be better suited to measure the complexity of objects in the physical world, a remaining challenge is the reliable measurement of the actual decompression time, which is difficult to single out from all other ongoing processes running at the same time in the operating system. Li et al. (2009) exemplified the L-Z complexity method, Lempel and Ziv (1976), for the measurement of landscape fragmentation on simulated and real urban land use data. Here, a binary map is transferred into a sequence of characters, which is then scanned from left to the right and the complexity counter is increased when a new subsequence of consecutive characters is encountered. Compared to the traditional set of landscape metrics they found it to be a better approach to quantify landscape fragmentation. These examples illustrate that the holistic concept of complexity may be used to describe the amount and the degree of information content inherent to the visual representation of image objects.

Entropy: The concept of entropy is used in many application fields from information theory to compression technology, medicine, economics, sociology, and mathematics. In ecology entropy has been used to measure the abundance of species in a given landscape, Chao and Shen (2003). Landscape fragmentation leads to decreased connectivity, increased isolation, and habitat loss, is often caused by anthropogenic activities, and has a large impact on biodiversity, Sala et al. (2000). Forman and Godron (1986) demonstrated that heterogeneity (pattern) and entropy could be considered as equivalent terms. Entropy has been used by Johnson et al. (2001) to characterize multi-resolution profiles of fragmentation pattern in landscapes. Joshi et al. (2006) used Shannon entropy as a measure of disorder of forest patches in remote sensing imagery over Northern India. The relation of entropy and several patch size metrics is discussed in detail in Bogaerts et al. (2005). They showed that fragmentation can be considered as the deviation from a contiguous space and its assessment is related to patch size diversity measurement. In thermodynamics, entropy describes the degree of disorder of the molecules in a gaseous system. In this paper we will transfer this idea of disorder into spatial geometry on raster images where we can think of entropy as a measure of the spatial disorder of target pixels and as such representing fragmentation. A concise overview on many aspects of spatial entropy can be found in Batty (1974).

This paper aims at illustrating normalized and quantitative methods to describe the overall degree together with the spatial distribution of fragmentation. Such normalized indicators have the additional benefit to quantify temporal changes in fragmentation over a given image and enable a direct comparison of fragmentation when comparing different test sites.

2. Methodology

This section describes three methods to quantify fragmentation. They are implemented in the free software collection GuidosToolbox, Vogt (2014), and available under the menu entry Image Analysis - Fragmentation. In general, the input image is defined as a categorical map (pseudo-binary raster mask of data type byte) with the assignment: 0 byte – missing data (optional), 1 byte – background, and 2 byte – foreground. Resulting images show normalized fragmentation values of the entire image as well as its spatial distribution.

2.1 Contagion:

Via an initial pop-up window, the user can specify the size of a square (kernel) window, which is then overlaid over each pixel of the input image, the metric is calculated for the area of the window, and the result is reassigned to the center pixel of the overlaid window in the output image (moving window approach). The contagion metric is calculated from the cell|cell adjacency values (edges). It is called P22 because we evaluate the proportion of edges having foreground on both sides ($2b|2b$). Here, we define N as the total number of edges between all pixels in cardinal directions, and the subset n as the number of edges that have foreground on one side or the other. All edges ($N-n$) that do not have foreground on either side are excluded in the metric calculation. If there are missing cells in the (kernel) window, the edges involving missing cells are not included. As a result, the total number of edges is less than N and the total number of edges involving foreground may be less than n , if missing cells are adjacent to foreground cells. P22 estimates the conditional probability that, given a foreground pixel, its neighbor is also foreground. For example, if all edges in the window have foreground on both sides then $P22=1.0$. Fragmentation can now be defined as the complement to contagion.

2.2 Complexity

Assuming fragmentation to be proportional to the amount of image information then it may be described via the degree of complexity of the image objects. For example, an image with a very simple object arrangement has little information content while a complex spatial distribution has more information content. The complexity of image objects is directly correlated to the compressibility of the image data. A suitable compression technology for this purpose must be lossless and spatially invariant to achieve similar results when processing the same image rotated by 90° . This requirement rules out many popular one-dimensional compression methodologies like gzip, which work on a line-by-line basis. In this study we chose the compression methodology jpeg2000 (2000), which includes a lossless mode based on a 2D discrete (bi-orthogonal) wavelet transform filter, Akansu and Haddad (1992). We can then calculate the ratio of compressed to uncompressed image file size. Furthermore, this ratio can be normalized by scaling it into the range of minimum (empty or fully filled image) and maximum (50% cover and random distribution) range of complexity. The calculation of the complexity via compression ratio cannot be performed on a per-pixel basis but requires a statistically meaningful neighborhood area, or tile size. Ideally, the compression ratio for this tile size should then be calculated in a moving window approach over the entire image. However, in GuidosToolbox the complexity is calculated by averaging calculations using box size tiles of 50 pixels and 33 pixels and starting from the center of the image. Both box sizes provide sufficient samples to be statistically relevant and cover different sized neighborhood tiles at different corner locations in the image. After averaging the entropy calculations for the two tile size setups a final smoothing filter is applied in order to return a spatially contiguous per pixel distribution in the image. Internal tests have shown that this approach yields similar results compared to the slower performing moving window

computation. On the special case of the checkerboard the other two methods provide 100% fragmentation while this method will provide counter-intuitive results here: due to the highly regular arrangement of the pixels in a checkerboard the compressibility will be high and thus wrongly suggest a low fragmentation value. However, in real life a checkerboard constellation is unlikely to be found. The main purpose of this option is to show how the very simple concept of compressibility/complexity may be used as a quantitative fragmentation measure.

2.3 Entropy

Starting from the classical definition of entropy in information theory, Shannon (1948), we define the discrete set of probabilities P_i as the probability that the difference between two adjacent pixels is equal to i and \log is the base 2 logarithm (Equation 1):

$$H = - \sum_i P_i \log(P_i) \quad (1)$$

The original entropy definition has been implemented in many ways and it is important to distinguish the above definition of P_i from other commonly used indicators such as Shannon's diversity index or the evenness index (where P_i is the proportion of *species*) and variations of contagion indices (where P_i is the proportion of different type of pixel edges). In short, in Shannon's original concept P_i refers to percentages of species classes in categorical maps, as defined in the species diversity literature. In contrast, here we investigate differences between cell values in all 8 directions (that is, the values of i). This is meaningful because raster images exhibit continuous variables where their magnitude has meaning. While the entropy in the edge-type evenness, Wickham et al. (1996), is derived from the attribute adjacency table, the spatial entropy here is calculated on spatial tiles and assuming 8-connectivity for the foreground pixels of a categorical map. For a given amount of foreground area, an image with a single compact foreground object has minimum entropy while the entropy reaches its maximum value when the given area is split into the maximum number possible and dispersed over the entire image. Maximum entropy is thus found for a checkerboard distribution. These two boundary conditions define the possible range of entropy (fragmentation) in the image. As with complexity, the entropy calculation cannot be conducted on a per-pixel basis but requires a statistically meaningful neighborhood area. For the same reasons outlined before the moving window approach is replaced by averaging calculations using box size tiles of 50 and 33 starting from the center of the image and applying a final smoothing filter.

3. Results and Discussion

Figure 1 shows the normalized fragmentation as a function of contagion, complexity, and spatial entropy. In addition, a summary of the fragmentation values is calculated for the entire image, the foreground area only, and the range of foreground pixel values in the image. Together with the file name of the analyzed image these values are displayed in the tile bar of the user interface. In the panel below the viewport the user can activate and specify two threshold values in order to show geographic areas having low, medium, and high fragmentation values. Threshold values segmenting the range of fragmentation values may be selected from a drop-down menu or specified interactively.

For all three approaches low fragmentation values are found in extended compact forest patches and high values in the upper right and lower central part of the image. By definition, contagion is more focused on the direct local neighborhood and for this reason exhibits highest values in areas being dominated by isolated small patches. The results for complexity and entropy are similar, with entropy having a slightly larger range of values. Even though their algorithmic nature is completely different this result is not surprising because both

concepts are descriptors of the information content inherent in the image objects. Compared to entropy the computation times are 1.5 for contagion and 3.0 for complexity. The batch-processing option of GuidosToolbox for automated mass data processing provides only the options entropy and contagion. Complexity has been dropped here in favor of entropy, which performs faster, provides similar results, and provides the correct value of 100% fragmentation for the checkerboard case.

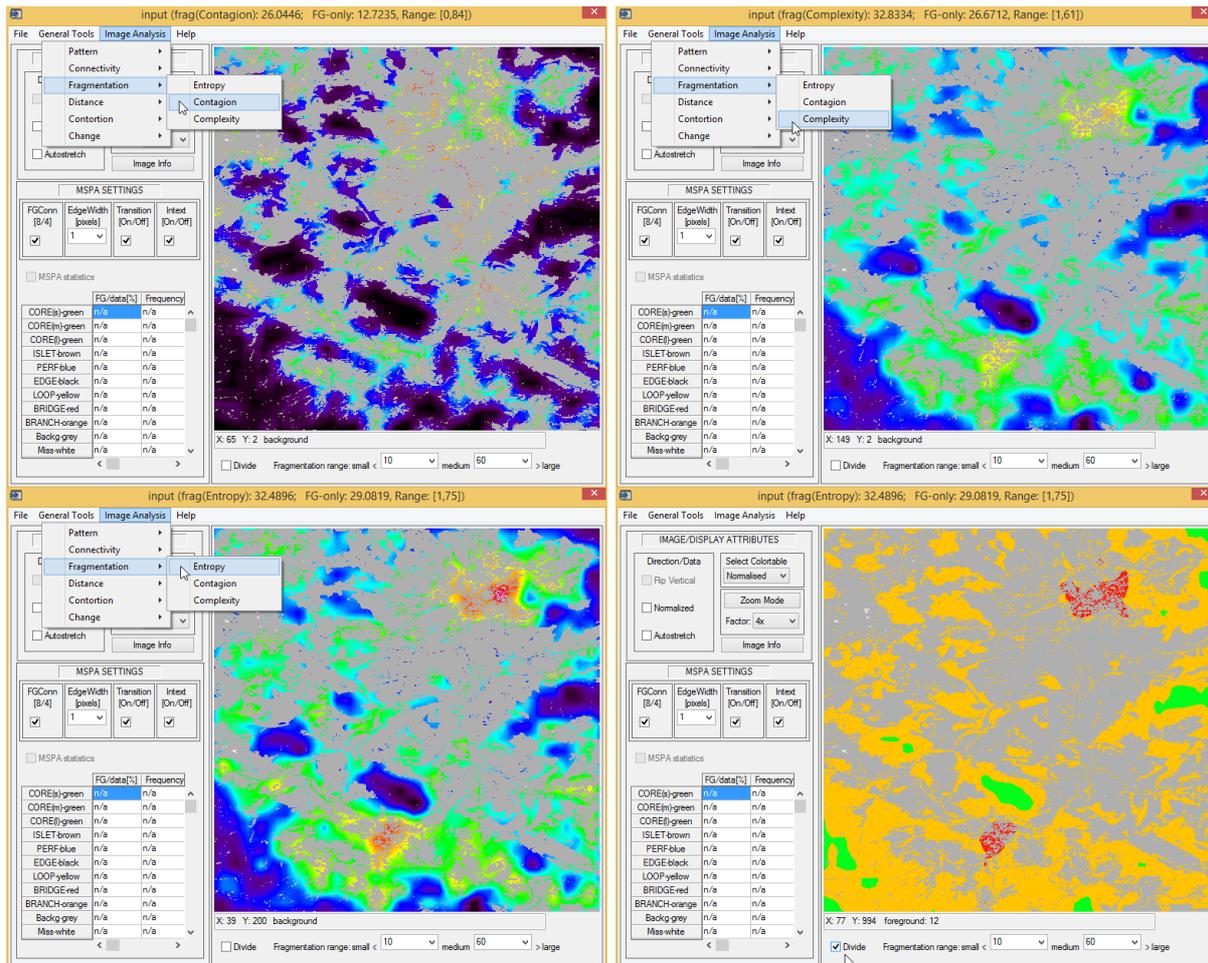


Figure 1. Normalized fragmentation calculated on a binary forest map and expressed as a function of contagion (top left), complexity (top right), and spatial entropy (bottom left). The tile bar provides a summary of fragmentation values for the entire image, the foreground pixels only, and the range of values of the foreground pixels. The image at the bottom right shows the segmentation of the bottom left image into areas of low, medium, and large fragmentation values driven by two user-selectable thresholds settings.

Despite the fact of a common agreement on the generic meaning of fragmentation this widely used term triggers different associations in different research and application fields. The perception and scientific significance of fragmentation may vary depending on the thematic field, the species under study, the importance attributed to different aspects of image properties, and even the interpreter conducting the analysis. The lack of a concise quantitative definition for fragmentation may simply originate in the fact that it is virtually impossible to summarize all these aspects in a consistent way. For this reason we do not attempt to solve this riddle with a single solution but instead describe fragmentation as a function of different holistic concepts. Depending on the task entropy may be a more appropriate descriptor in one case while another researcher may favor contagion as a suitable descriptor for fragmentation

because his interest is more focused on isolated features. For this reason we provide more than one option in GuidosToolbox to measure fragmentation in digital images and further holistic approaches may be added in the future.

The focus of this study was to provide neutral assessment schemes based on geometric concepts only and resulting in repeatable, normalized fragmentation values at pixel level. Instead of addressing and summarizing the many different aspects of fragmentation individually the proposed fragmentation measures apply holistic approaches using generic concepts of contagion, complexity, and spatial entropy. It is important to note that any subsequent statistical analysis in fact requires a reliable quantitative assessment of fragmentation. Moreover, normalized indices permit not only a concise state assessment on a given site but also the direct evaluation of fragmentation when comparing different sites. Finally, and when investigating temporal changes, the suggested indicators allow localizing and especially quantifying the extent of the fragmentation changes on the map.

4. Conclusion

This study described three methodologies resulting in normalized fragmentation metrics describing both, the overall degree as well as the spatial distribution of fragmentation of any categorical land cover type map in a digital raster image. Besides highlighting hotspots of changes the proposed indicators permit measuring, and thus quantifying the progress in biodiversity and landscape planning projects. The optional user-driven division into low, medium, and large values may be helpful for landscape planners to quickly pinpoint hotspot areas of high fragmentation for further observation or immediate treatment. The application of the suggested tools is facilitated through their provision in a free software package.

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